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Submitted to Academy of Financial Services for October 9-10 2010, Denver CO

Sequence Risk: Managing Retiree Exposure to Sequence Risk Through Probability of Failure Based Decision Rules

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Hypothesis and Executive Summary

Hypothesis

Decision rules may be developed that use Monte Carlo Probability of Failure (POF) (aka ruin) rates to:

1) Monitor the exposure to the sequence risk that down, and up, markets impose on portfolios in distribution, and;
2) Decide on actions that contain the Probability of Failure rate within reasonable ranges through both portfolio allocation and withdrawal adjustments over time.

Executive Summary

- Past first generation withdrawal research has emphasized maximizing withdrawal rates with little focus on *relative* probability of failure (POF) where POF rates are compared between strategies to determine true effectiveness of the strategy proposed.
- Withdrawal rates alone do not tell a complete sustainable distribution story. The distribution period affects the sustainable withdrawal rate as does the portfolio allocation. The Probability of Failure is the common denominator useful for true comparison of withdrawal rates over any time period or asset allocation.
- Probability of Failure surfaces stack consistently over each other like layers of an onion. Comparison of these POF surfaces, and their shift between strategies, begins the process of understanding how effective a strategy is as compared to another strategy and as compared to fixed baseline benchmark POF surfaces.
- The methodology used in this study used annual revisits for switching the strategies used. Monthly revisits most likely would refine the findings. Annual revisits were used initially here simply to demonstrate the concepts, which are more important than any strategy at this initial stage, for purposes of broadening the perspective on sustainable distributions into three dimensions, transitory states, and all retiree states existing simultaneously.
- A basic three dimensional baseline withdrawal landscape exists which any retiree may map into during that transitory state snapshot. By implementing a strategy, with the most common strategy researched to date being spending retrenchment, a comparison between strategies is possible by comparing the transitory states of each strategy. For example, a fixed unchanging distribution dollar amount may have a particular POF whereas reducing the withdrawal amount reduces the POF.
- The findings here refine how much of a change is required to bring POF back into a reasonable range with both time and allocation considerations.

Brief Overview.

Past research has interpreted static simulations to represent the future, implied by the term "initial," versus recognition that all the variables are dynamic over time. The problem with such a viewpoint, as demonstrated by the market upheavals of 2007 and 2008, is that markets often don't behave as they were projected to behave. Therefore, the concept in this paper is to ignore, or drop off, past years that have gone by for the retiree and "start over" at the current time and re-

evaluate the three variables involved in retirement withdrawals: current portfolio value, current portfolio allocation, and current withdrawal rate, over the fourth variable of time *remaining*. This concept is captured using the term "current" as opposed to the term “initial.” This becomes an iterative process as the retiree continues to age.

The perspective of this research shifts from previous research in two ways: 1) the focus of clients is on dollar amounts rather than sustainable rates; and 2) the financial planner adopts a probability of failure (POF) approach rather than the traditional withdrawal rate (WR) approach given the reality of the WR changing over time.

Published research focuses on the sustainability of retirement withdrawals over various periods of time in order to ascertain a given withdrawal rate percentage’s (WR%) probability of failure (POF), probability of sustainability, or probability of ruin. All these probabilities are essentially synonymous. In general, past research has tended to focus on maximum sustainable rates (MSR) as suggested by Liu, et al (2009) and Stout and Mitchell (2006), combined with withdrawal rate-based decision rules by Guyton (2004), Guyton and Klinger (2006), Pye (2008), Stout (2008), and Mitchell (2009). If the retiree continues to strive to maximize withdrawals as they age, the result is also an exposure to sequence risk. This problem is exacerbated by unpredictable significant market movements (Black Swans) as discussed by Mandelbrot and Hudson (2004) and Taleb (2007). The challenge then is to manage both withdrawal maximization and sequence risk when adverse events occur (a market decline or unexpected large withdrawal required – both having a similar effect on sustainability).

The problem with a maximization of withdrawal rate approach is that the withdrawal rate alone does not directly inform the observer whether ruin is imminent for the time required, hence the issue of sequence risk and various approaches to establish decision rules to manage this exposure. Research by Frank and Blanchett (2010) suggests that probability of failure (POF) may be a useful tool to evaluate a retiree’s exposure to sequence risk at any given moment in time. This is because POF ranges remain consistent over time while WR% increases as the retiree’s distribution period shorten as they age. The withdrawal rate variable is time-dependent. The probability of failure variable is time-independent. See Figure 1.

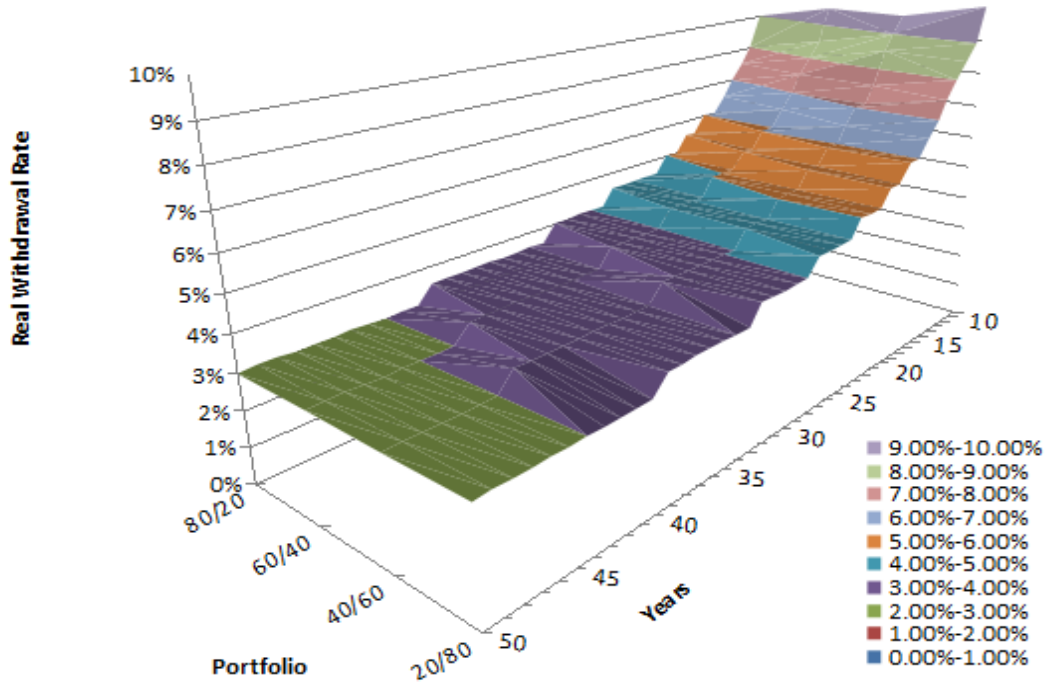


Figure 1. Probability of Failure of 0% to 5% with changing withdrawal rate and portfolio allocation over time.

There exists a three dimensionality to the withdrawal rate problem which the authors are investigating in this paper. Much of the past research has involved static allocation simulations which the authors represent in this paper as research along the x-axis of Figure 1. This is versus a dynamic allocation approach which investigates changes along the y-axis of Figure 1. Such a y-axis investigation is represented with a recent paper by Garrison, Sera and Cribbs (2010).

All points graphed in Figure 1 have *the same approximate probability of failure rate* (0-5%). The data for Figure 1 comes from Blanchett and Frank (2009) which has been expanded here to include the lower withdrawal rate regions of the landscape. Additional data points would serve to smooth Figure 1.

Most of the existing research focuses on changing WR% along the x-axis with some research looking at Asset Allocation along the y-axis of Figure 1. However, the research along the y-axis consists of fixed allocations for the duration of the distribution time (z-axis), essentially keeping the allocation question within a single plane in the figure, that plane being a static asset allocation. In other words, little research (Garrison, Sera & Cribbs, 2010 is a recent exception) has explored the effect of changing the Portfolio Allocation *within the simulation* along the y-axis of Figure 1, as has been done for changing Withdrawal Rates *within the simulation* over time.

Higher Probability of Failure threshold landscapes would map out higher on the x-axis compared to the 0-5% threshold illustrated, since corresponding withdrawal rates are higher. In other words, the 0-5% landscape is the lowest probability of failure landscape. Note that as distribution time shortens, withdrawal rates may increase, all with a similar approximate exposure to probability of failure.

This paper explores the dynamics of Probability of Failure as a function of time, for both Withdrawal Rates (WR%) and Portfolio Allocations (PA). Essentially, the retirement distribution problem is a function of four variables: Time (t), Withdrawal Rate (WR%), Portfolio Allocation (PA), and Probability of Failure (POF). The hypothesis for this exploration is that POF based decision rules may be developed that may warn a retiree that their retirement withdrawals during adverse returns sequences may put their distributions at risk of depletion within their lifetime. With such a warning, these same decision rules may suggest a method to adjust either the portfolio allocation and/or withdrawal amount to avoid running out of money before running out of life.

The point of reference for Guyton type decision rules, based on an *initial* withdrawal rate, is by definition a reference to some point in the past. This paper will shift the point of reference for decision rules into the future by using a future oriented reference point for the time function and will also use *current* withdrawal rates as opposed to initial withdrawal rates.

Asset classes.

Returns for the analysis are based on five assets. Returns are from January 1926 until December 2009. All returns are converted into "real returns" (i.e., are adjusted for inflation), where the definition of inflation is the increase in the Consumer Price Index for Urban Consumers, data obtained from the Bureau of Labor Statistics.

Cash: 30 Day T-bill
Bonds: Ibbotson Associates Long-term Corporate Bond Index
Domestic Large Equity: S&P 500
Domestic Small Cap Equity: Ibbotson Associates US Small Stock Index
International Large Equity: Global Financial Data Global ex USA Index from January 1926 until December 1969 and then the MSCI EAFE from January 1970 until December 2009.

The portfolios for the analysis are made of two primary components a Cash/Fixed piece and an Equity piece. The Cash/Fixed piece is 25% Cash and 75% Bond. The Equity piece is 50% Domestic Large Equity, 25% Domestic Small Equity and 25% International Large Equity. For example, a 60/40 portfolio (60% Equity and 40% Cash/Fixed) would have a 10% Cash allocation, a 30% Bond allocation, a 30% Domestic Large Equity allocation, a 15% Domestic Small Equity allocation, and a 15% International Large Equity allocation.

This paper will use data that excludes skewness and kurtosis. A subsequent project will duplicate this paper's methodology using data to incorporate the impact of non-normal distributions.

Time sequencing and simulation periods.

The research in this paper is divided through the perspective of time, both time sequencing and simulation periods. Time sequencing refers to the stochastic generation of returns. Simulation periods refers to the length of time each stochastic simulation is run.

Simulation distribution periods, from *each* point in Figure 2 are 10-year, 15-year, 20-year, 25-year, 30-year, 35-year, and 40-years. Evaluation of research by Blanchett and Frank (2009) suggests that longer periods tend to begin to "flatten" out - it is the shorter periods that have the most changes when you look at graphs and figures. The purpose of this step in the investigation is to parse out where POF landscapes are the same (i.e., 0-5% landscape, >5-10% landscape, >10-15% landscape, etc.) in order to determine at what POF landscape changes begin to emerge as decision and comparison points.

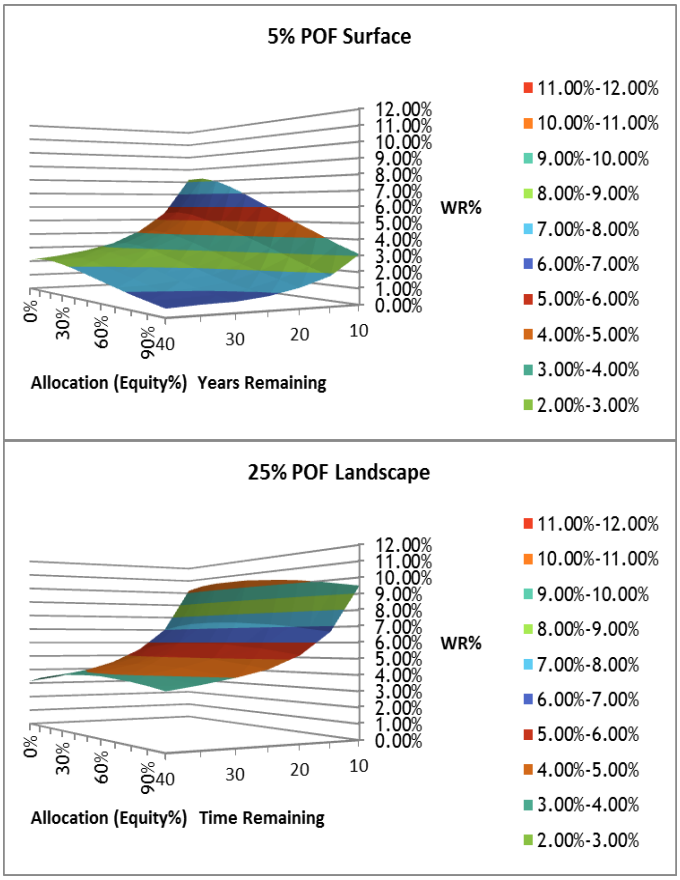
Since sequence risk is, by definition, a result of a time series, the investigation should look at a sequence of portfolio returns over time. For example, a period of market decline which simulates market deterioration over time, as well as a subsequent recovery. It is this market decline, combined with continued withdrawals over time, which generates sequence risk and exposes a portfolio undergoing withdrawals to adverse effects.

We tend to think in terms of simulations and project that single result (withdrawal rate) "forever" into the future. If we run a simulation at time T_1, we get a certain result. Nevertheless, things change as time changes. Now, at time T_2 where the market has declined 5% and we run another simulation, we get another result. The same at time T_3 when the market is now, for example, down 10% relative to T_1, etc. The same continues to happen as long as the market declines at each time period T_n; and the reverse occurs when the market recovers. So POF based decision rules not only need to look at POF decision points while the market deteriorates, but also at decision points to reverse the process as the market recovers.

Therefore, there exists a symbiotic relationship between time and percentage decline/recovery and the corresponding change in POF. Isolating the effects of market declines and evaluating both a change in withdrawal amounts (WR%) and portfolio allocation (PA) over time begins to develop a series of three dimensional "data clouds" of which one such surface was illustrated in Figure 1 previously.

Step 1. Maximum *fixed withdrawal amounts* at a particular POF with no decision changes to serve as a baseline for subsequent comparison.

For Step 1, a 10,000 run Monte Carlo generator was built in Microsoft Excel. The distribution is assumed to be taken from the portfolio at the beginning of each year. All returns are in "real" (inflation-adjusted) terms, so that a constant withdrawal amount is assumed to be taken from the portfolio during each year in retirement. The "success" of a portfolio or withdrawal is calculated by determining how many portfolios had positive values at the end of the year. A positive value would indicate the portfolio was successful for that year. Withdrawal rates are tested in .05% increments from 0% to 25%, in .10% increments from 25% to 50%, and in .25% increments from 50% to 100%.


5% POF Surface
12.00%
11.00%
10.00%
9.00%
8.00%
7.00%
6.00%
5.00%
4.00%
3.00%
2.00%
1.00%
0.00%
WR%
0%
30%
60%
90%
40
30
20
10
Allocation (Equity%)
Years Remaining
11.00%-12.00%
10.00%-11.00%
9.00%-10.00%
8.00%-9.00%
7.00%-8.00%
6.00%-7.00%
5.00%-6.00%
4.00%-5.00%
3.00%-4.00%
2.00%-3.00%
25% POF Landscape
12.00%
11.00%
10.00%
9.00%
8.00%
7.00%
6.00%
5.00%
4.00%
3.00%
2.00%
1.00%
0.00%
WR%
0%
30%
60%
90%
40
30
20
10
Allocation (Equity%)
Time Remaining
11.00%-12.00%
10.00%-11.00%
9.00%-10.00%
8.00%-9.00%
7.00%-8.00%
6.00%-7.00%
5.00%-6.00%
4.00%-5.00%
3.00%-4.00%
2.00%-3.00%

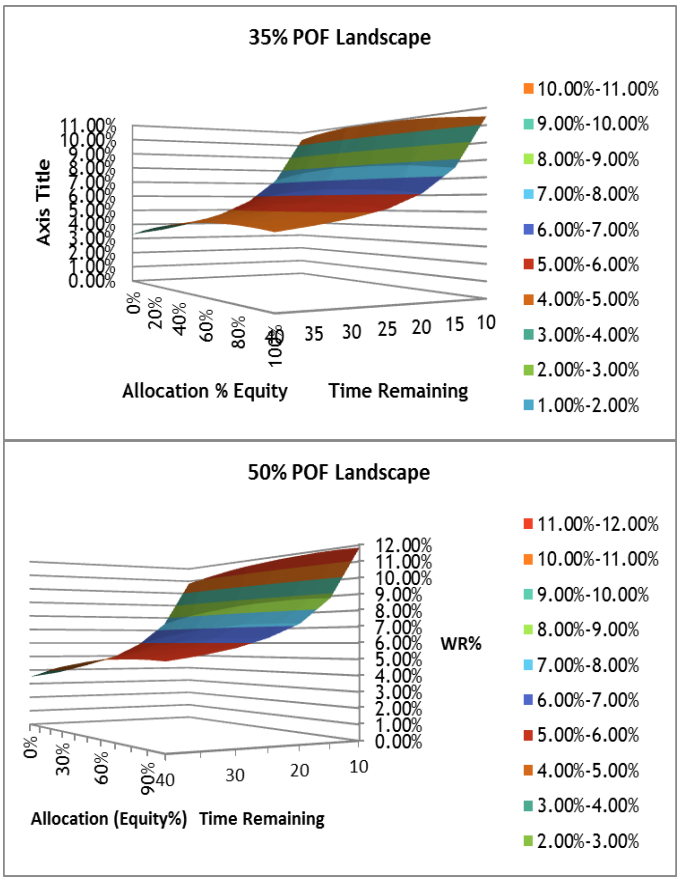
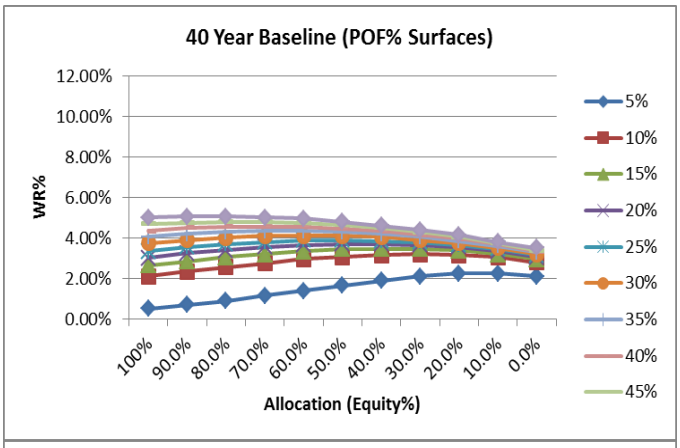
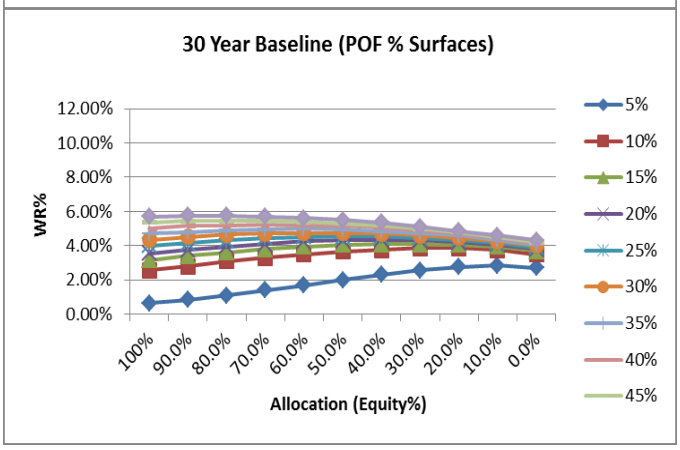


Figure 2. Probability of Failure landscapes for Fixed withdrawals.

Figure 2 illustrates how different degrees of probability of failure (POF) layer three dimensionally on top of lower POF values. The data for this and following figures are derived by a focus on where the different POF surfaces emerge. The 5% POF surface lies under the 25% POF which in turn is below the 50% POF surface for all time periods and asset allocations. Figure 2 represents fixed withdrawal rates and forms a three dimensional baseline for later comparison of different strategy responses to market sequences.


40 Year Baseline (POF% Surfaces)
WR%
12.00%
10.00%
8.00%
6.00%
4.00%
2.00%
0.00%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
5%
10%
15%
20%
25%
30%
35%
40%
45%


30 Year Baseline (POF % Surfaces)
WR%
12.00%
10.00%
8.00%
6.00%
4.00%
2.00%
0.00%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
5%
10%
15%
20%
25%
30%
35%
40%
45%

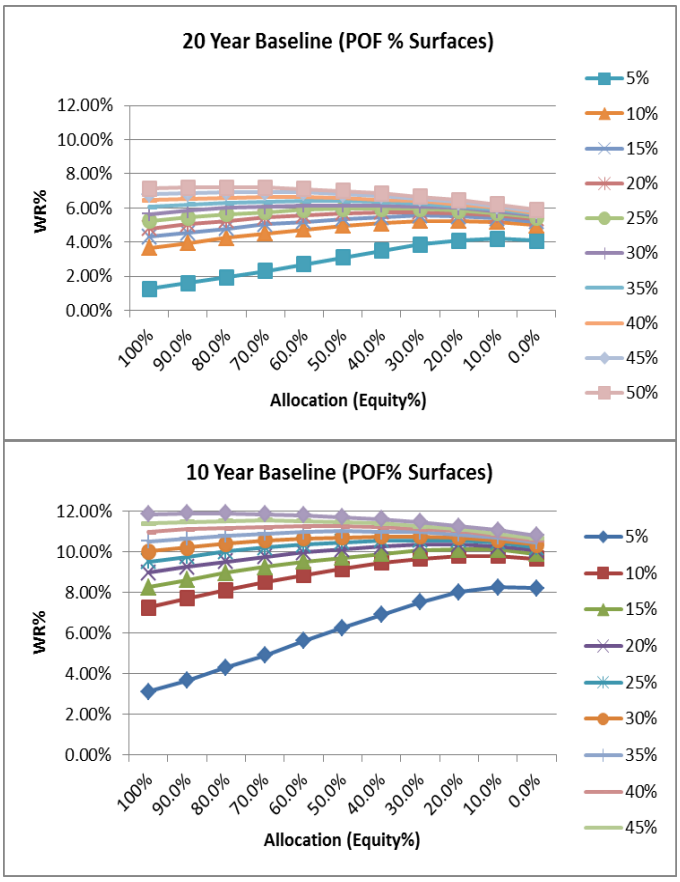


Figure 3. Time slices through Figure 2.

Figure 3 are slices through figure 2 at various times to form time-planes. This plane view through the POF surfaces allows visualization of how the withdrawal rates, with the same POF value, shift up as distribution time shortens for the time periods 40, 30, 20 and 10 years. Consistent tendencies also emerge as to asset allocations.

Notice the consistent wide dispersion of POF surfaces for high volatility/high equity allocations relative to the narrow dispersion of POF surfaces for low volatility/low equity allocations. The implications of this dispersion will be discussed in step 2 below.

Introducing the Concept of Transitory States.

This Baseline data set represents starting a fixed withdrawal at each moment in time. Although the simulations are run in this manner, in life each point is transitory depending on factors beyond the retirees' control: 1) the decreasing time remaining for the retirees' distribution (reference point for distribution periods for each retiree is some set, retiree specific, future target end age (Blanchett/Frank 2009, Frank/Blanchett 2010) and 2) the portfolio value which is a function of market forces as a result of bad or good market sequences.

Each data point in 3D represents the transient state defined by the results of a simulation run. Each time a simulation is run for a retiree the POF results of that simulation would plot somewhere within the three dimensional space defined by time, withdrawal rate and portfolio

allocation and would represent the current transient state of that retiree at that moment in time. The fundamental withdrawal rate formula is Withdrawal Rate (WR%) = Annual Dollars Withdrawn ($Y) divided by Portfolio Value ($X) (WR% = $Y / $X). The withdrawal rate has an inverse relationship to the portfolio value given a set annual dollar withdrawal. Visualize a retiree at one point in time and withdrawal rate, and then one year later at another point in time and a different withdrawal rate simply because their portfolio value changed with time. The initial conditions that existed a year earlier no longer exist; only the current conditions exist and thus apply for this retiree going forward until the next transitory state is evaluated. For example, a retiree with 31 years of distribution time remaining may have a *current* withdrawal rate of 4% at that transitory state with a POF between 25 to 30 percent, depending on asset allocation. One year later, market forces on the portfolio value may have changed the withdrawal rate to 5% moving the POF above 35%, depending on portfolio allocation. Of course, this is a fixed, unchanging withdrawal amount and portfolio allocation and the market forces required to increase the withdrawal rate for a conservative portfolio would be greater than those for a more aggressive portfolio. However, the point is that this retiree has a 5% transitory withdrawal rate for this example.

Now, should the market conditions not improve over this retirees' remaining lifetime, i.e., a negative sequence begins at this point in time, the transitory higher POF may indeed prove true; or worse, the fixed unchanging withdrawal amount may continue to push the retirees' withdrawal rate higher resulting in higher POFs as time goes by. All retirees at the 30 year point with a 5% withdrawal rate would have a corresponding POF based on their asset allocation regardless of how long ago they may have started their retirement. This is true of any point in time.

This transitory nature due to time and sequence risk, either good or bad, is the impetus for evaluating the *relative* transitory states of other possible withdrawal strategies. *If* a retiree changed their allocation and/or their withdrawal amount at any point in time, how would this change their POF?

Baseline Comments.

A few points at the end of Step 1. First, stopping the illustrations at 50 percent POF is purely arbitrary and selected merely as a POF that the authors deem as a sufficient boundary that most retirees probably would find as an unacceptable risk, i.e., this simply is an arbitrary point to demonstrate what happens up to this point along the POF spectrum. Second, the POF landscapes exist along the full spectrum of POFs between 0 to 100 percent and the authors have chosen 5 percent increments for illustration purposes. Of course, interpolation is possible between the research points along any of the axis'. Should a plot of all possible data points, or possible states, be constructed a three-dimensional "data cloud" would emerge, through which the illustrated landscapes (POF), horizontal slices (WR) and vertical planes (time) would intersect for each individual retiree.

Following is a brief review of the variables, and their axis as depicted on the three-dimensional graphs, with their relationships to each other:

- Annual Dollars Withdrawn ($Y): $Y = $X multiplied by WR%; where Δ$Y (the new withdrawal amount) is a result of the retiree making a change to their annual dollar withdrawal.
- Current Portfolio Value ($X): $X = $Y divided by WR%; where Δ$X is a result of portfolio value changes in proportion to the PA, which are driven by changes in the markets, as well as their withdrawal for the period.
- Withdrawal Rate (WR%) is depicted on the **Z-axis**: WR% = $Y divided by $X; ΔWR% is a result of Δ$Y divided by Δ$X.
- Portfolio Allocation (PA): is depicted on the **Y-axis**. ΔPA is a result of the retiree making a change to their Portfolio Allocation.
- Time (t): t is depicted on the **X-axis** and ticks down to represent distribution time remaining for the retiree as they age. Δt is what generates a change in the markets which then has an effect on the WR depending on the various market weights within the PA.
- Probability of Failure (POF) is depicted as the various data points based on PA, WR%, and t. Possible POF data points possible correspond to the smallest change in each of the three variables, PA, WR% and t. ΔPOF is a result of either, or both, Δ$Y and ΔPA; which are the *only* two variables that the retiree has any direct control over. By changing $Y and or PA, the retiree also indirectly effects WR%.

This paper extends previous research in order to answer the following questions: What are more refined decision rules for a retiree? At what WR does this retiree need to reduce their withdrawal amount? At what WR may this retiree increase their withdrawal amount? More importantly, is a single "withdrawal-rate-adjustment rule" true of any POF? Is this true of any time period? Is this true of any asset allocation?

The objective of the subsequent steps in this research is to separate out two strategies over which a retiree has real control: either 1) changing their asset allocation, hence exposure to market volatility; or 2) changing their withdrawal amount, hence the effect of negative dollar cost averaging affects. The final step is to determine what decision rules may emerge to aide distribution decisions as the retiree continues to encounter sequence risk, both positive and negative in nature, throughout retirement. In summary, the focus of step 1 above is simply to establish a baseline upon which to compare the subsequent research steps 2 and 3 in order to develop a decision rule regime for step 4.

Methodology for the next two steps:

For Step 2, the Step 1 generator was modified so that each year in each run the distribution withdrawal dollar amount would change based on the previous annual return of the portfolio. If the portfolio return was greater than .5 standard deviations away from the average (e.g., a portfolio with an average expected real return of 5% and a standard deviation of 10% would need to experience a real return of 10% (5% + .5 x 10%) or greater) the dollar withdrawal would be increased by 3%. If the portfolio return was less than .5 standard deviations away from the average (e.g., a portfolio with an average expected real return of 5% and a standard deviation of 10% would need to experience a real return of 0% or less) the dollar withdrawal would be decreased by 3%. This modification was made to every year in each of the 10,000 runs. The portfolio allocation was not changed in this step, only the annual dollar withdrawal amount.

For Step 3, the Step 1 generator was modified so that each year in each run the equity allocation of the portfolio would changed based on the on the previous annual return of the portfolio. If the portfolio return was greater than .5 standard deviations away from the average (e.g., a portfolio with an average expected real return of 5% and a standard deviation of 10% would need to experience a real return of 10% or greater) the equity allocation would be decreased by 10%. If the portfolio return was less than .5 standard deviations away from the average (e.g., a portfolio with an average expected real return of 5% and a standard deviation of 10% would need to experience a real return of 0% or less) the equity allocation would be increased by 10%. The dollar withdrawal was not changed in this step, only the allocation.

Step 2. Time-varying *withdrawal amounts within simulations* based on a decision rule that maintains a constant POF. Withdrawal amounts, rather than withdrawal rates, are presented because clients tend to focus on withdrawal amounts rather than withdrawal rates. Blanchett and Frank (2009) first reviewed the dynamically changing of withdrawal rates when the withdrawal *amount* was adjusted. This step reformats the research approach to focus on POF landscapes to find the associated withdrawal rate.

Similar 3D graphs, comparable to Figure 2, may be constructed to compare the POF surfaces when a withdrawal strategy has been applied to a retiree. In step 2, the authors' strategy adjusts the withdrawal dollar amount (Δ$) in response to negative (bad), or positive (good), market sequences. Rather than draw graphs similar to Figure 3, the authors illustrate the shift in POF landscape surfaces between the baseline strategy (step 1) and change allocation strategy (step 2).

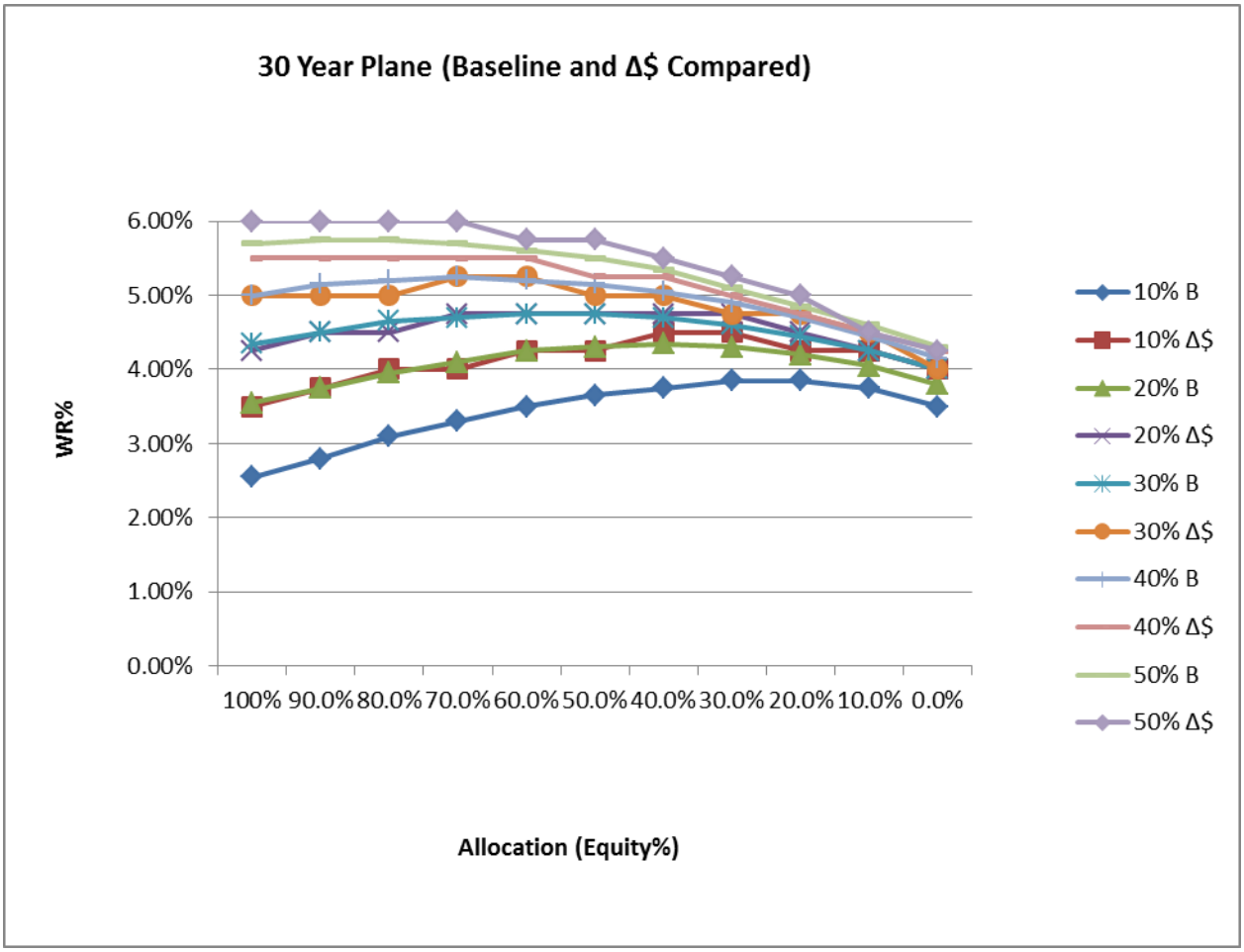
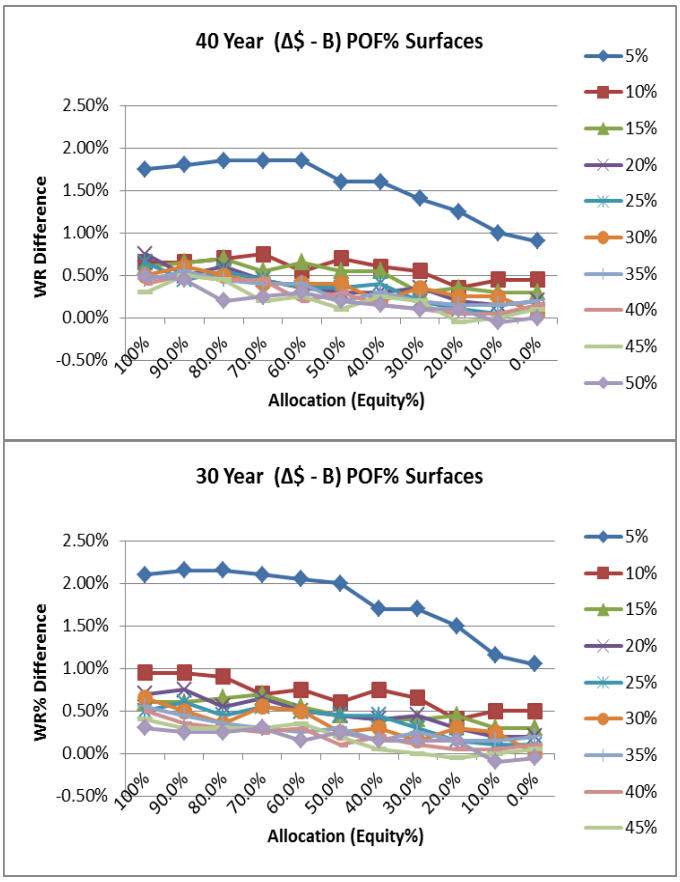


Figure 4. Comparison between POF surfaces between Baseline (B) and Changing Withdrawal $ Amount (Δ$) for 30 year plane.

Figure 4 is similar to Figure 2 except this time comparing the shift in withdrawal rates between the changing dollar amount (spending retrenchment/expansion) strategy and the Baseline strategy in order to illustrate the differences between a Baseline strategy (B) and a changing withdrawal dollar amount strategy (Δ$). Notice that the POF% landscape surfaces shift up; i.e., the corresponding WR% is different for each POF% surface. Example, the 10% B surface lies below the 10% Δ$ surface, etc.

Figure 5 below is similar to Figure 3 and results from subtracting the Baseline withdrawal rate from the change of withdrawal amount, thus illustrating the differences, or shift, between the POF% surfaces for the 40, 30, 20, and 10 year planes. Figure 5 compares the transitory states, by illustrating the difference between the baseline strategy and the changing dollar withdrawal strategy. Once again, notice that all the similar POF surfaces shift upwards, e.g., comparing 10%B to 10%Δ$, etc. and that the difference between strategy narrows as the POF increases. The WR% values in Figure 5 are the result of subtracting the B WR%'s from the Δ$ WR%'s. ($WR_{\Delta\$}$ - WR_B). The convention used from this point forward in this paper for WR_B , $WR_{\Delta\$}$, (and $WR_{\Delta PA}$ in step 3) is that the variable represented is the withdrawal rate (x-axis) in all cases and is understood as such; therefore the WR term is dropped and that the subscript is used for better readability.


40 Year (Δ$ - B) POF% Surfaces
WR Difference
2.50%
2.00%
1.50%
1.00%
0.50%
0.00%
-0.50%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
5%
10%
15%
20%
25%
30%
35%
40%
45%
50%
30 Year (Δ$ - B) POF% Surfaces
WR% Difference
2.50%
2.00%
1.50%
1.00%
0.50%
0.00%
-0.50%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
5%
10%
15%
20%
25%
30%
35%
40%
45%

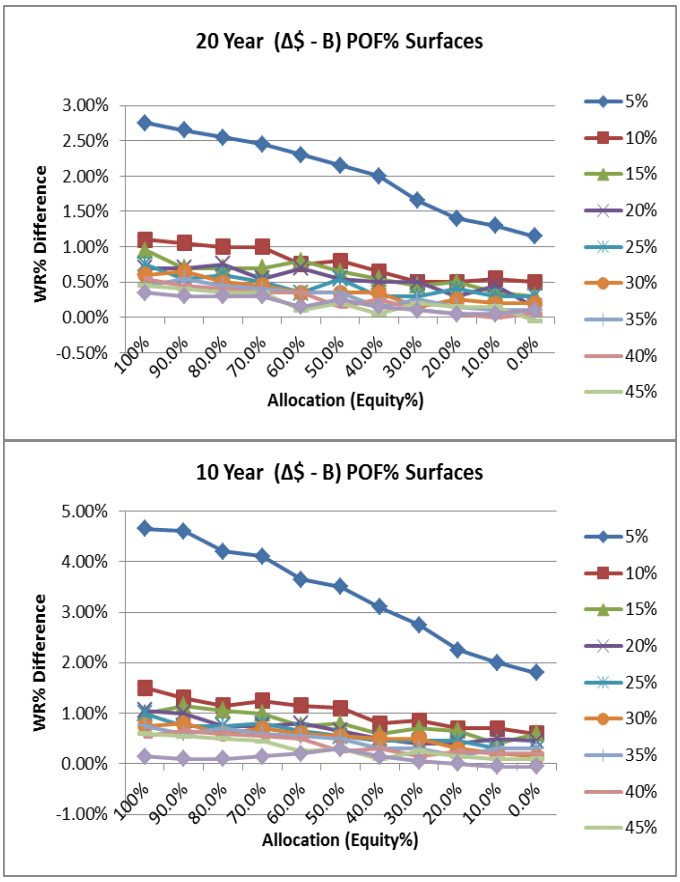
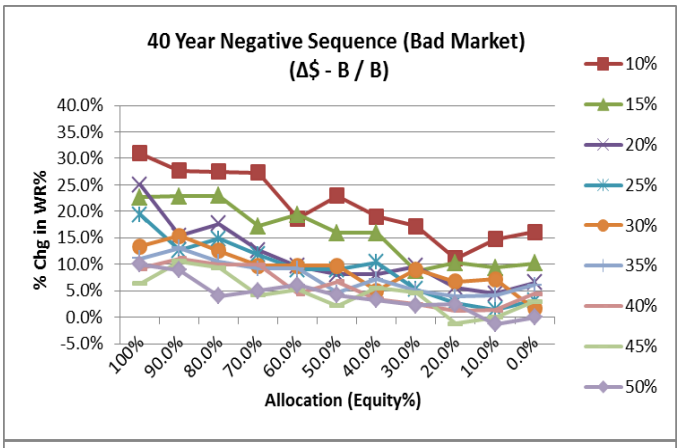
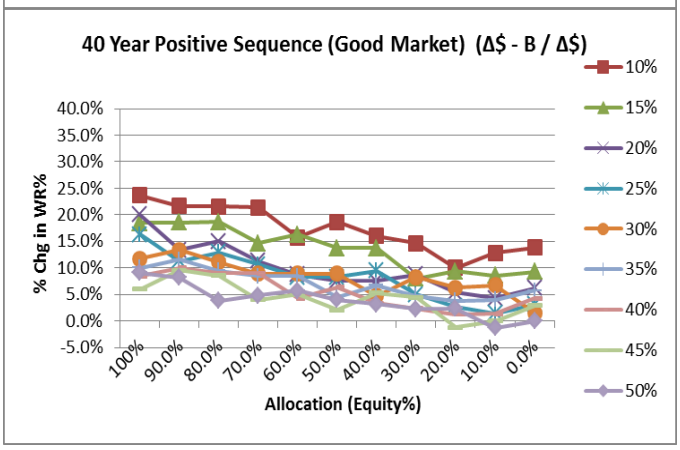


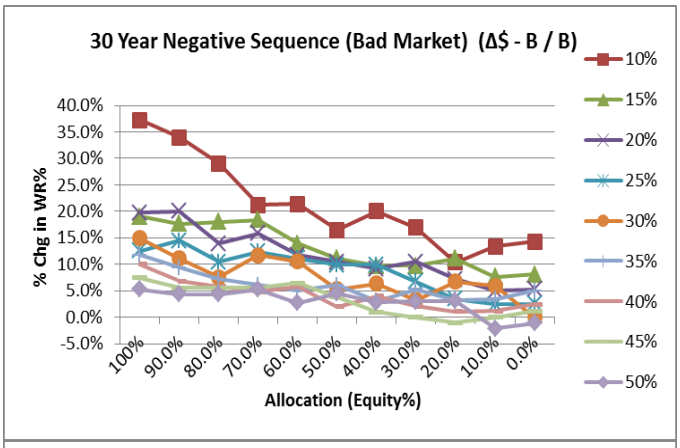
Figure 5. Comparison of WR differences between Baseline (B) and Δ$ strategies.

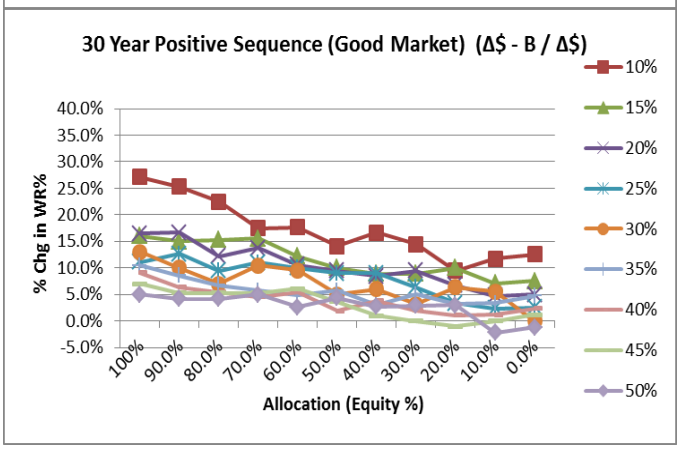
The differences between WR values in Figure 5 represent the change in the WR value itself. For example, a 1% difference in Figure 5 may be between a WR of 4% and 5%. Figure 5 illustrates the differences between similar POF surfaces in order to illustrate the effect of time on those differences, i.e., 40 year, 30, 20, and 10 year planes. Observe that the time effect is small for the differences between the POF surfaces (Figure 6) while the time effect is large for the change in WR% values themself (Figures 3 and 4) suggesting more sensitivity of WR% as POF increases which demonstrates that the magnitude of the shift decreases as the POF increases. *Thus, the higher the POF, the less the effect of a strategy change.*

The differences in Figure 6 below represent the percentage magnitude of change when referenced to either the lower WR% value (left side-market deteriorating) or higher WR% value (right side-market improving). For example a 1% WR change from 4% to 5% would represent a 25% change in value for a rising WR (bad market-left side), or a 20% change in value for a declining WR (good market-right side). The comparison is between similar surfaces. 10%B to 10%Δ$, etc.


40 Year Negative Sequence (Bad Market)
(Δ$ - B / B)
% Chg in WR%
40.0%
35.0%
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
10%
15%
20%
25%
30%
35%
40%
45%
50%


40 Year Positive Sequence (Good Market) (Δ$ - B / Δ$)
% Chg in WR%
40.0%
35.0%
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
10%
15%
20%
25%
30%
35%
40%
45%
50%


30 Year Negative Sequence (Bad Market) (Δ$ - B / B)
% Chg in WR%
40.0%
35.0%
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
10%
15%
20%
25%
30%
35%
40%
45%
50%


30 Year Positive Sequence (Good Market) (Δ$ - B / Δ$)
% Chg in WR%
40.0%
35.0%
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
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Allocation (Equity %)
10%
15%
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25%
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40%
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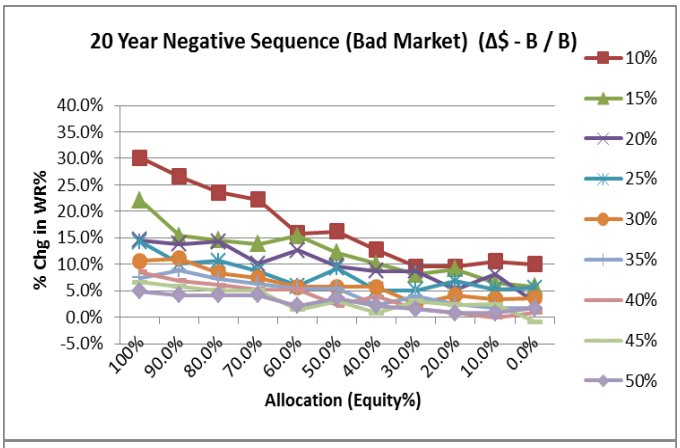
20 Year Negative Sequence (Bad Market) (Δ$ - B / B)
% Chg in WR%
40.0%
35.0%
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
100%
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40.0%
30.0%
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Allocation (Equity%)
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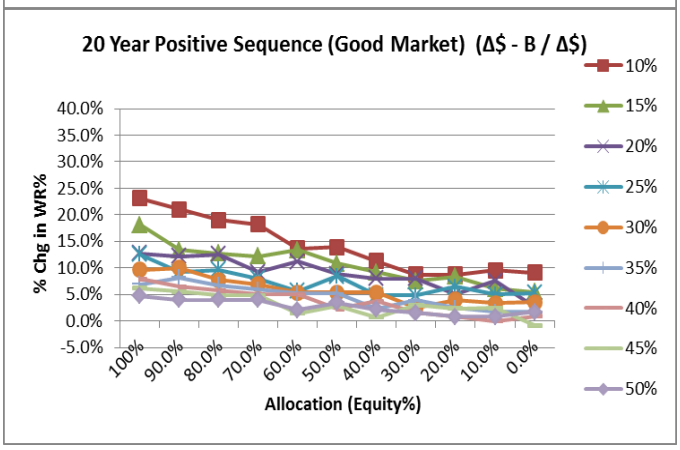
35%
40%
45%
50%


20 Year Positive Sequence (Good Market) (Δ$ - B / Δ$)
% Chg in WR%
40.0%
35.0%
30.0%
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
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0.0%
Allocation (Equity%)
10%
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40%
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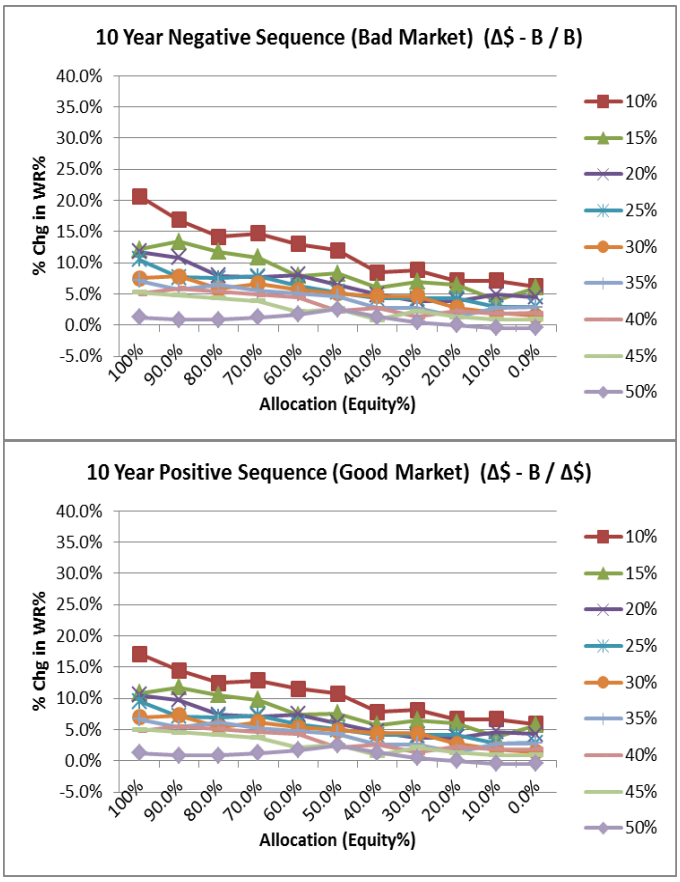


Figure 6. Left half: Negative Sequence (Bad Market) (Δ$ - B / B); Right half: Positive Sequence (Good Market) (Δ$ - B / Δ$) for 40, 30, 20, and 10 year periods.

The left side of Figure 6 represents a negative, or bad, market sequence for 40, 30, 20 and 10 year periods for a strategy of withdrawal dollar amount adjustment. The right side of Figure 6 represents a positive, or good, market sequence for the given time period. Negative sequences are derived by comparing WR between strategies: Δ$ - B / B; Positive sequences are derived by comparing WR between strategies: Δ$ - B / Δ$. The fundamental withdrawal rate formula is WR% = $Y / $X with an inverse relationship between the withdrawal rate and portfolio value. Since WR% and POF are positively correlated, the direction WR% is taking through transitory states provides a signal as to what POF is also doing, either increasing or decreasing inversely with portfolio value.

The tolerable change of the withdrawal rate (the withdrawal amount compared to the portfolio value) varies between less than 35% for bad market sequences (100% equity) to less than 15% (100% bonds). For example, for 30 years, 4% Baseline WR, and 80% equity allocation (Figure 4) has a 20% POF; if the portfolio value declines in value due to a negative market sequence such that the current withdrawal rate is now 5%, then in an unchanging Baseline scenario the POF would be just under 40%. By adjusting the withdrawal amount by 3% (the magnitude of adjustment in this study), the POF is reduced to 30% (assuming additional future adjustments).

Notice, the degree of dispersion is again narrower for lower equity allocations relative to higher equity allocations for both negative and positive market sequences. Thus, a smaller withdrawal

rate change should trigger a retrenchment/increase in the withdrawal dollar amount for lower equity allocations as compared to higher equity allocations. There is no single withdrawal rate change value since the magnitude of the change varies by asset allocation and by time. In general though, the required degree of the percent change in withdrawal rate is a few percent less for positive market sequences relative to negative market sequences.

Across the timeline, the compression of the %chg in WR% curves suggests that the withdrawal rate should be adjusted *until the POF is less than 30%* (as opposed to the 30% WR% mentioned two paragraphs ago). For POF rates higher than 30% the %chg in WR%" is much smaller, or more sensitive to portfolio value changes, regardless of allocation, and thus additional withdrawal amount adjustments would be necessary. It is this 30% POF value that is more critical than the 30% generalized "%chg in WR%" value. The results in Figure 6 show that the amount of WR% change is not a constant, but changes depending on time, portfolio allocation and the POF with which the retiree is comfortable.

Waiting for markets to improve the portfolio value later in order to improve the POF (i.e., depending on the historical improvement of markets) may not be a prudent assumption in that those market improvements may be long in coming. The methodology developed here measures the POF in order to adjust the withdrawal rate prudently sooner, rather than later, in response to what is actually occurring. Monitoring the improvement of POF during moments of positive sequence risk provides a signal to increase the withdrawal dollar amount as well.

Note that the "%chg in WR%" value as used in this paper does *not* refer to an *initial* withdrawal rate. Rather, the use here is in reference to the last time the withdrawal rate was measured as compared to the current measurement of the withdrawal rate, or comparison between two transient states. However, rather than thinking the withdrawal rate may change by 25% for example, the results here suggest that POF is a more important factor to consider. Note that for higher POF values, the withdrawal rate change is much less than for lower POF values; i.e., the 50% "%chg in WR%" values are consistently less than the 10% %chg in WR% values. The conclusion is that there is more room for withdrawal rate change for lower withdrawal rates than for higher withdrawal rates and that the POF provides the clue as to how much room for withdrawal rate change actually may exist. Therefore, saying that the withdrawal rate may change by 20% for example is misleading since the relative POF has not been considered in addition to the current withdrawal rate.

Step 2 Decision Rule Concepts for bad market sequences and, for good market sequences.

Since a retiree can only evaluate their *current* WR% they are essentially only able to evaluate their current transient state in terms of the Baseline figures. However, the results from step 2, changing amount of dollars withdrawn, can be overlaid and comparisons made as in Figure 6 above.

First general observation:

As time shortens, for each 10 years, the POF landscapes reduce (shift down) by 5%.
Shorter distribution periods are more sensitive to market sequences.

Second general observation:

The lower POF landscapes do most of the shifting where the higher POF landscapes do not. Retirees who already have a high *current* withdrawal rate are more sensitive to market sequences than those with a lower *current* withdrawal rate. The dividing line for market sequence sensitivity appears to be the 20% POF landscape.

Third general observation:

Good market sequence POF shifts are generally 5% less than bad market sequence shifts for distribution periods longer than 10 years. After retrenchment, spending may be increased sooner since a smaller withdrawal rate change is needed.

Step 3. Time-varying *asset allocations within simulations* combined with Baseline withdrawal amounts based on a decision rule that maintains a constant POF. These portfolio allocations were those chosen from above. Garrison, Sera and Cribbs (February 2009) evaluated dynamically changing the portfolio allocation during simulations using a trailing 12 month simple moving average as the portfolio switching signal. The authors here have instead used the change in the portfolio value itself as the signal to switch between portfolios as described in methodology above. The results that follow from the authors research will be compared to the baseline data in step 1.

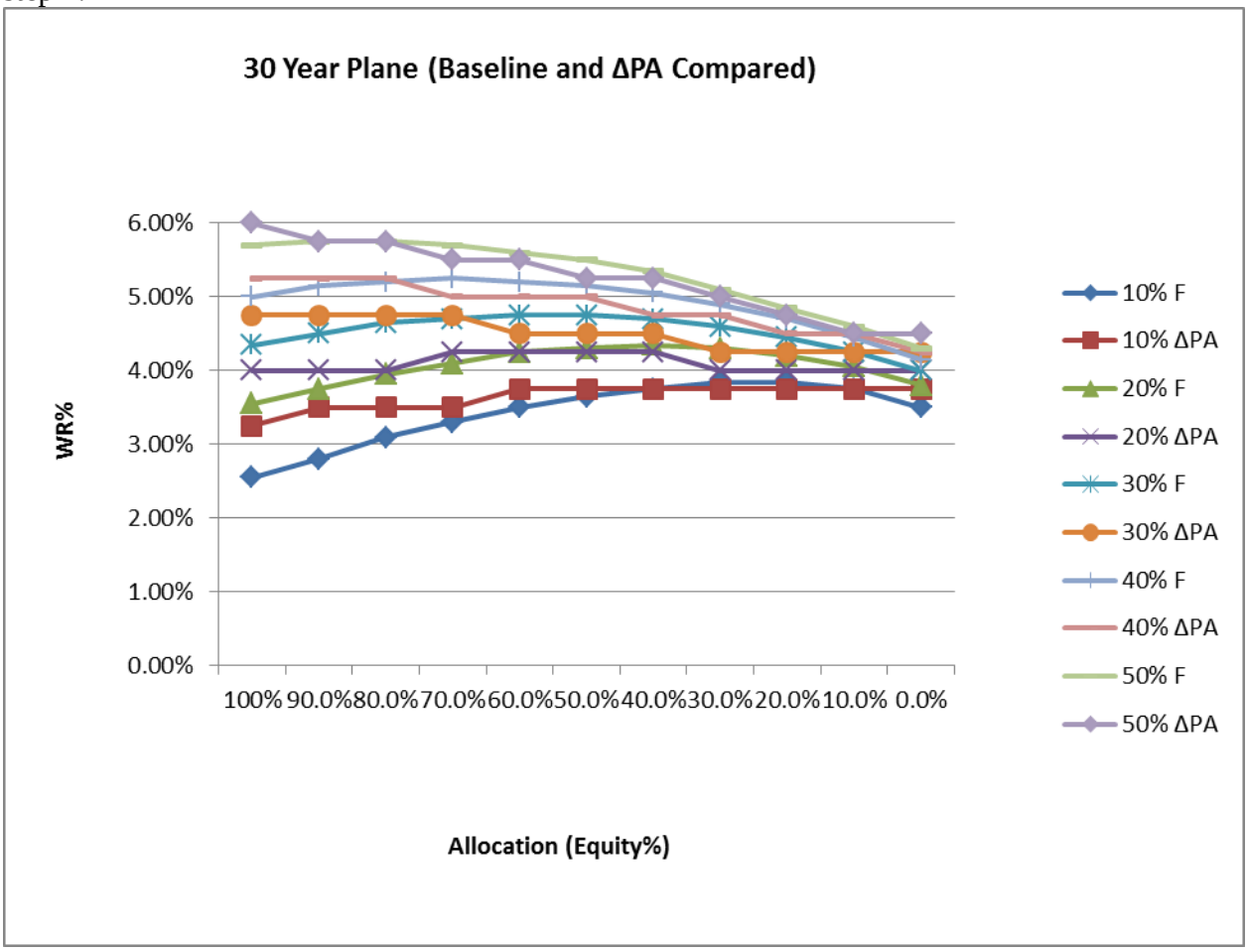
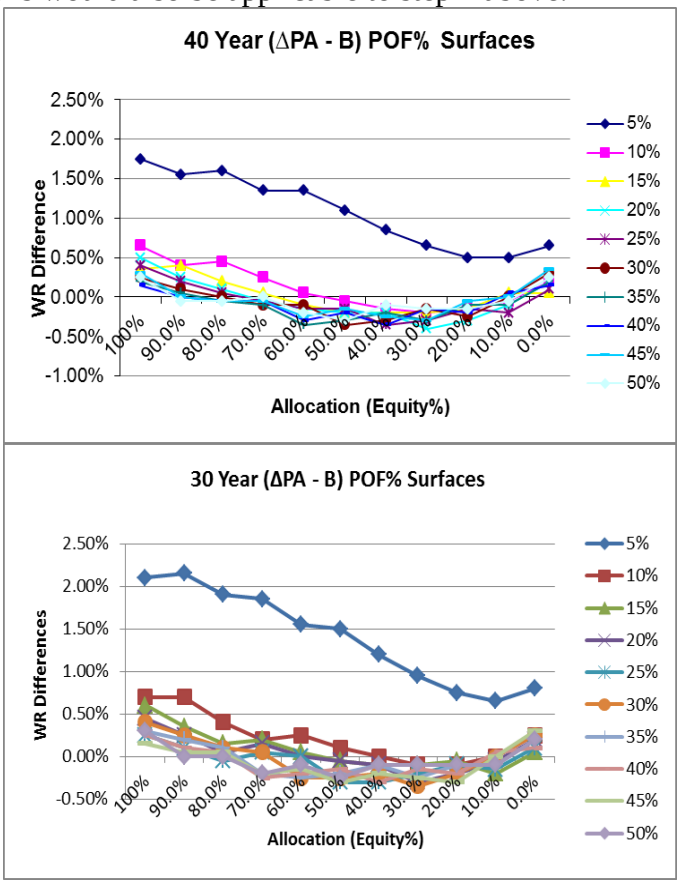


Figure 7. Comparison between POF surfaces between Baseline and Changing Portfolio allocation for 30 year plane.

Figure 7 illustrates this POF shift comparison for the 30 year time plane. This shift is similar to Figure 4, except this time the shift results from a different withdrawal strategy in response to sequence risk. Notice that the POF% landscape surfaces shift up; i.e., the corresponding WR% is different for each POF% surface. Example, the 10% B surface lies below the 10% ΔPA surface, etc. However, the shift is less dramatic for this strategy.

Notice the "shift" between withdrawal rates, illustrated in Figure 7, when comparing the baseline starting WRs to the delta Portfolio Allocation (ΔPA) starting WRs. This "shift" between possible withdrawal rates suggests several options for the retiree: 1) begin their withdrawals using the higher WR (ΔPA) with the intent to adjust their allocations based on what happens to their portfolio value over time; 2) begin their withdrawals using the lower WR (Baseline withdrawals) which provides a cushion necessary to absorb negative market events, in other words to allow the withdrawal rate to increase when markets pull the portfolio value down (hence withdrawal rate goes up through the inverse relationship) until the higher WR is reached which then suggests the point has been reached to begin a portfolio allocation strategy; or 3) some combination of these two. Note, these options would also be applicable to step 2 above.



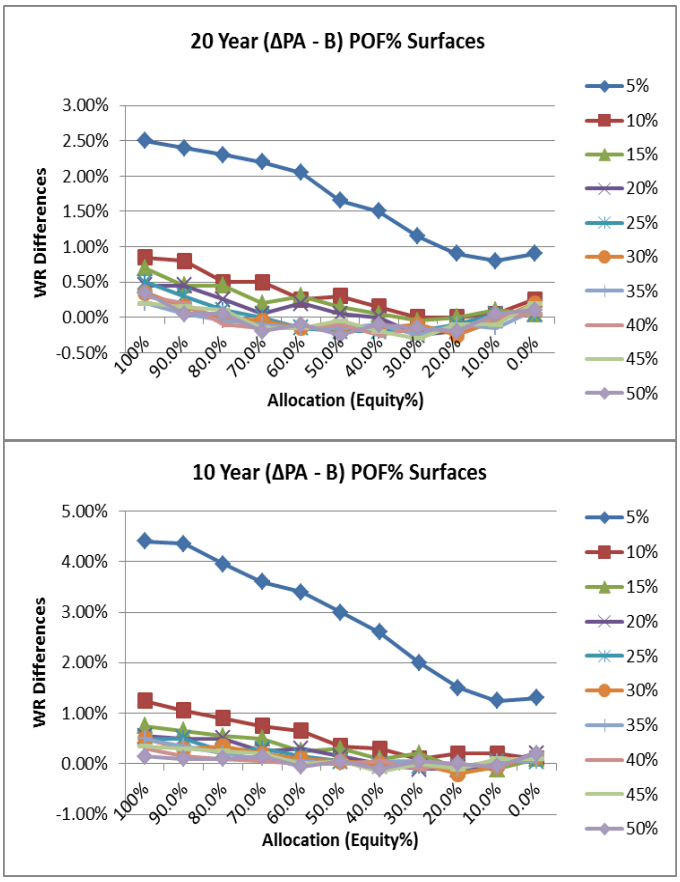
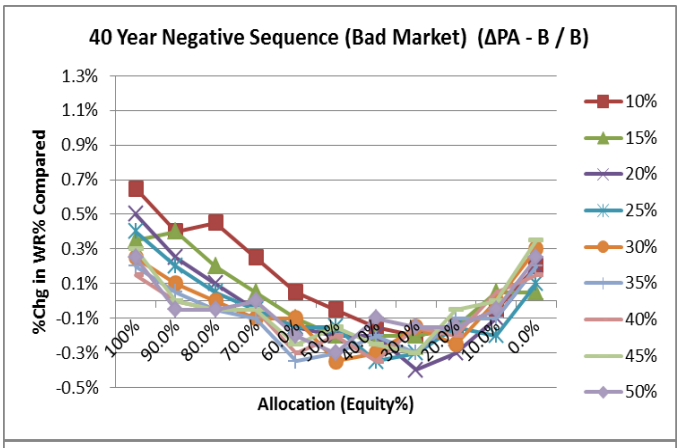
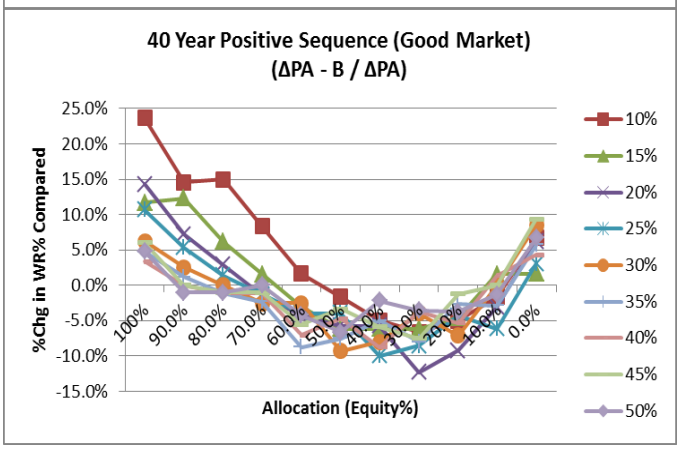
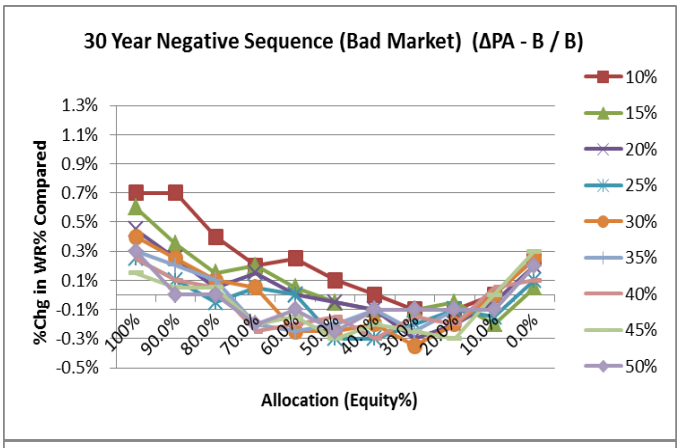


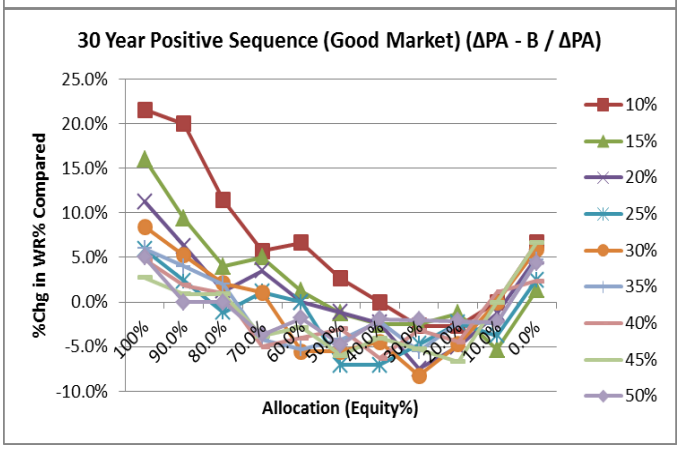
Figure 8. Comparison of WR differences between Baseline (B) and ΔPA strategies.

Figure 8 results from subtracting the Baseline withdrawal rate from the change in portfolio allocation withdrawal rate, thus illustrating the differences between the surfaces for 40, 30, 20, and 10 year planes. Figure 8 is similar to Figure 5 by comparing different strategies over various time planes. Figure 8, just like Figure 5, demonstrates that the magnitude of the shift decreases as the POF increases. *Thus, the higher the POF, the less the effect of a strategy change.*


40 Year Negative Sequence (Bad Market) (ΔPA - B / B)
%Chg in WR% Compared
1.3%
1.1%
0.9%
0.7%
0.5%
0.3%
0.1%
-0.1%
-0.3%
-0.5%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
10%
15%
20%
25%
30%
35%
40%
45%
50%


40 Year Positive Sequence (Good Market)
(ΔPA - B / ΔPA)
%Chg in WR% Compared
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
-10.0%
-15.0%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
10%
15%
20%
25%
30%
35%
40%
45%
50%


30 Year Negative Sequence (Bad Market) (ΔPA - B / B)
%Chg in WR% Compared
1.3%
1.1%
0.9%
0.7%
0.5%
0.3%
0.1%
-0.1%
-0.3%
-0.5%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
10%
15%
20%
25%
30%
35%
40%
45%
50%


30 Year Positive Sequence (Good Market) (ΔPA - B / ΔPA)
%Chg in WR% Compared
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
-10.0%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
10%
15%
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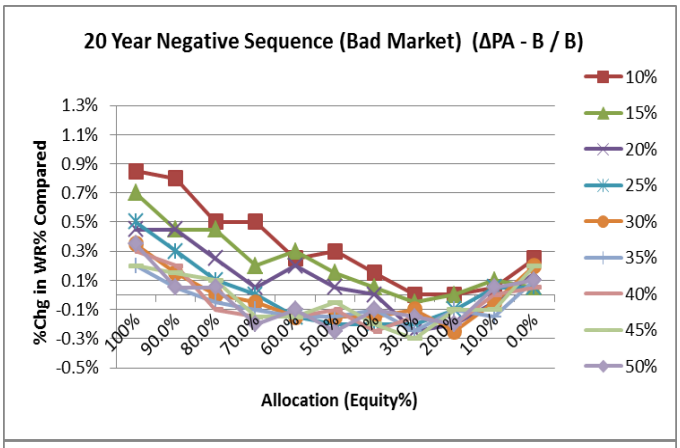
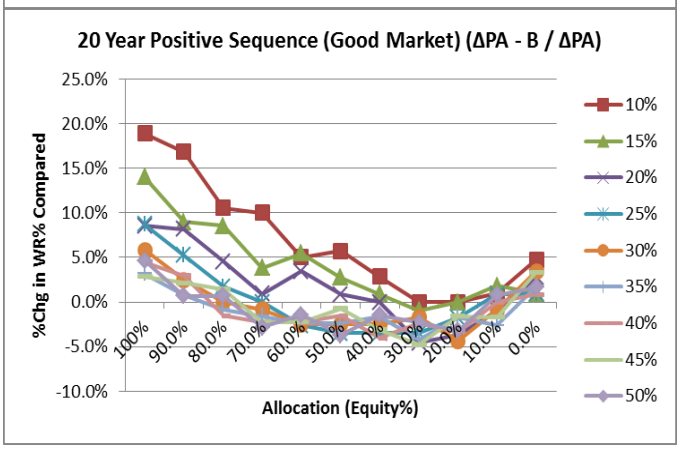

20 Year Negative Sequence (Bad Market) (ΔPA - B / B)
%Chg in WR% Compared
1.3%
1.1%
0.9%
0.7%
0.5%
0.3%
0.1%
-0.1%
-0.3%
-0.5%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
10%
15%
20%
25%
30%
35%
40%
45%
50%


20 Year Positive Sequence (Good Market) (ΔPA - B / ΔPA)
%Chg in WR% Compared
25.0%
20.0%
15.0%
10.0%
5.0%
0.0%
-5.0%
-10.0%
100%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
20.0%
10.0%
0.0%
Allocation (Equity%)
10%
15%
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25%
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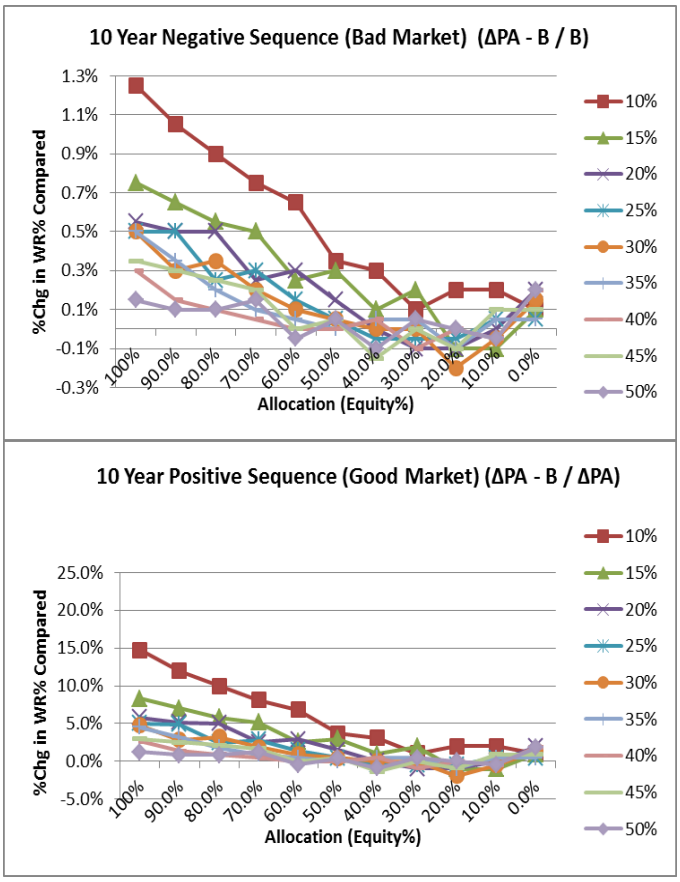

Figure 9. Left half: Negative Sequence (Bad Market) (ΔPA - F / F); Right half: Positive Sequence (Good Market) (ΔPA - F / ΔPA) for 40, 30, 20, and 10 year periods.

Figure 9 illustrates the amount of withdrawal rate differences between comparable POF surfaces during either negative sequence (bad markets-left side) or positive sequence (good markets-right side). Negative sequences are derived by comparing WR between strategies: ΔPA - F / F; Positive sequences are derived by comparing WR between strategies: ΔPA - F / ΔPA.

Recall that lower respective POFs correspond to lower respective WR% and vice verse.

The left side of Figure 9 represents a negative, or bad, market sequence for 40, 30, 20 and 10 year periods for a strategy of portfolio allocation adjustment. The tolerable change of the withdrawal rate (the withdrawal amount compared to the portfolio value) is typically less than 1%. For example, for 30 years, 4% Baseline WR, and 80% equity allocation (figure 4) has a 20% POF; if the portfolio value declines in value due to a negative market sequence such that the current withdrawal rate is now 5%, then in an unchanging Baseline scenario the POF would be just under 40%. The ΔPA POF is very close in this case so a portfolio adjustment would not enhance the retiree's POF.

The right side of Figure 9 represents a positive, or good, market sequence for the stated time period. Notice, the percent change in withdrawal rate is much larger, comparable to those of increasing the withdrawal dollar amount in step 2. However, that effect tends to disappear as the equity allocation is reduced to approximately 50 percent.

General conclusions about the strategy of changing asset allocation alone.

The general conclusion about changing asset allocation alone as a strategy to address sequence risk is that it is not effective as a stand alone strategy. The negative sequence, or bad market series (left side of Figure 9), %chg in WR% values are so small, or even negative (indicating the fixed strategy has higher WR%), that they are essentially the same as the baseline WR% values. Although the %chg in WR% values are large for the positive sequence, or good market series (right side of Figure 9), the dilemma is that this would imply allocating towards more aggressive allocations during growing markets and then *not* allocating towards more conservative allocations during declining markets; hence, over time the allocation would be the 100% equity allocation which is likely more volatility than the retiree is comfortable with. Thus, the allocation answer becomes more of a behavioral allocation comfort issue than it is as a stand alone sequence risk strategy.

The question remains; does an allocation change enhance a retiree's overall POF outcome if it is combined first with another strategy? In other words, does changing the allocation first and then change the withdrawal amount enhance the retiree's outcome? Further research is required using this comparative methodology to address combination strategies.

Additional research using this comparative methodology is also required to evaluate the effect of data skewness and kurtosis when comparing the shift in POF surfaces. The hypothesis is that POF based decision rules may keep the retirees distributions within the bounds where both normalized data, and non-normalized data are very similar and hence the application of effective strategies may keep the retiree away from the fat tails in their experience even though the market is experiencing such events.

The objective of the research project is to establish Probability of Failure based decision rules to evaluate the retiree's *current* exposure to sequence risk in declining markets. Conversely, these general observations would aid the retiree decisions during both recovery from market declines and/or "normal" increases in market values over time; in other words, when exposure to sequence risk is favorable.

Conclusions.

The purpose of this paper is to introduce a method to demonstrate the concepts, which are more important than any strategy at this initial stage, for purposes of broadening the perspective on sustainable distributions into three dimensions, transitory states, and all retiree states existing simultaneously. The authors introduce the methodology of comparing a proposed strategy against the baseline fixed data in order to determine the efficacy of the proposed strategy and to determine refinements and possible breakpoints for decisions using the proposed strategy.

Time, withdrawal rate, and portfolio allocation are the same for each graph. What changes is a shift in the probability of failure surfaces as a result of a strategy change. These strategy changes may be compared and used strategically in response to either negative or positive sequences of the market and that market's effect on the retirees' portfolio value.

The working hypothesis consists of taking a current snapshot of where the retiree lies within the three dimensions mapped out by the landscape suggested by earlier work in Figure 1 above. As time passes and the distribution period decreases, the *current* conditions of market value, relative withdrawal amount and portfolio value would map a new location in the landscape. Each simulation is a review that illustrates the current point within the landscape at each moment in time. This is not a set-and-forget approach to the use of simulations. Rather, it suggests a revisit of the current conditions when significant market events, or sequences, necessitate change.

Subsequent research is needed on integrating model based rules with that of retiree behaviors during both adverse and positive sequence risk periods.

Bridge between past research and research in this paper.

Past distribution research has focused primarily on three factors: withdrawal rate, distribution period, and portfolio allocation; all essentially static in that initial conditions are set and a probability of failure (or success) is determined. A dynamic perspective comes from the recognition that time is not a constant (every single person, including retirees, travels through time at the rate of one second per second) and thus the distribution period is not constant. The other factors are also not constant. By shifting perspective and focus on the Probability of Failure variable that does run through the time as a constant, and recognition of the transitory state concept, it becomes possible to begin to evaluate different withdrawal strategies and compare them to each other. This strategy comparison thus begins to illuminate possible courses of action a retiree may take as positive or negative event sequences occur.

A Note on Perspective.

A measure of probability of success, ruin or failure, should not be misinterpreted by advisers as a measure of risk. Risk is a measure of the consequences of events while probability is a measure of likelihood of those events. When events are negative in nature, the probability of adverse consequences goes up, and vice versa. Essentially another view and definition of sequence risk. This paper shifts perspective to one that considers multiple retiree transitory states that *all* coexist simultaneously with each state defined by time and the market's effect on portfolio values *relative to* the withdrawal amount of each retiree.

Using probability, as a measure in and of itself, may lead to an over allocation of stock holdings for longer time periods simply due to an inherent trend in the long term stock returns data. The authors here simply suggest that probability of failure, and its trends either up or down over time, may be utilized as a tool to assess exposure to sequence risk as portfolio values rise or fall. Adverse events must lead to a re-evaluation, as would positive events. The authors evaluated methods to address this sequence risk exposure through the lens of re-evaluation of the retiree's probability of failure, as time passes and events unfold, while ignoring what the retiree was able to withdraw in the past. In other words, the perspective is to forget and reset as time progresses. With such a perspective, whether data contains skewness or kurtosis is mute since the real purpose of using POF is to establish decision rules in advance of such potential occurrences, to use when market conditions present their effect on the retiree's portfolio value.

Expecting a model or simulation to predict what is right around the corner is unrealistic. Kurtosis, skewness, black swans, bubbles, etc are parts of uncertainty. Rather than try to build a model that may try to predict future events, arguably difficult if not impossible, the authors have developed strategies to address market sequences as they occur, both positive and negative. The authors suggest that simulations provide a window into what adjustments should be made. Once actual events occur, revisiting the simulations again over the remaining distribution time period may suggest new adjustments for the retiree based on where they currently are within the three-dimensional "data cloud. The objective of review as events happen is to re-evaluate and make adjustments based on the new information and current situation. No simulation or model may be developed that may accurately predict precisely what the future holds. This does not invalidate the use of the simulation. It simply requires that simulations be re-run as events unfold. Probability of failure is a method to evaluate the current exposure to adverse, as well as favorable, events. Arguably, no predictive metric may be available. However, Probability of Failure is an effective management tool for sequence risk.

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